Methods of Conversion of Stability of Femtosecond Stabilized Mode-locked Laser to Optical Resonator

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Abstract— **In this work we propose three methods of a conversion of RF clock stability to the length precision using fs stabilized laser via Fabry-Perot by three proven conversion techniques. One mode stabilized lasers could provide optical frequency reference. Stability of such a laser could be transferred to length etalon represented by optical interferometer. We present using of Fabry-Perot interferometer as a length etalon referenced to fs stabilized optical comb. The train of femtosecond pulses produces equidistant optical frequencies separated by repetition frequency and shifted by an offset frequency. Both frequencies are stabilized according to widely used f-to-2f stabilization method to RF frequency standard, i.e. atomic clocks represented by GPS controlled signal with relative stability down to 10^{-15}. First method of conversion is based on a simple assumption that the femtosecond laser light is transmitted only when a free spectral range (FSR) of Fabry-Perot cavity (FP) is a multiple of the repetition frequency of the laser. Precise length is inversely proportional to the repetition frequency and fixed to the position of pulse to pulse interferogram's centroid. In the second method FP length is chosen arbitrary to increase the FSR of the FP and only few frequencies of optical laser are transmitted and detected. FP length could be stabilized to any transmitted laser line. The third method is based on the frequency lock of DFB laser line to one of the frequency lines of femtosecond laser and by fixing the beat frequency between the DFB laser and the frequency synthesizerâĂŹs line. Length of FP is stabilized to DFB laser line.**

I. Introduction

Metrology of optical frequencies in these days often deals with frequency synthesis through pulsed femtosecond mode-locked lasers [1]. Value of repetition rate of generated pulses determines (in the frequency domain) spacing of discrete coherent components of the whole supercontinuum. The spectrum of the supercontinuum has profile of a comb. The device that generates the stable comb spectrum is called an optical synthesizer. It can convert stability of radio frequency (RF) repetition frequency into light spectral domain and vice versa. Therefore the synthesizer is considered as a very modern metrology tool because it bridges large gap between optical and radio frequency bands. The main application of the synthesizer is a comparison of stabilities between ultra-stable rf generators based on microwave atomic clocks (i.e. Rb, Cs) with optical frequency standards like lasers stabilized by molecular, atomic, or ions transitions (i.e. I_2, Yb^+ or Hg^+) [2].The relative stability of optical frequency standards is measured by stabilized femtosecond laser comb [3]. The definition of the one meter is based on either the time of the flight of light in the vacuum or through precise optical frequency of laser [4] referenced to interferometer. The standard method for conversion of optical frequency stability to the length stability is based on the transmission through an optical resonator [5]. Stabilized mode-locked lasers present sources of pulses with very stable spacing and new application in the field of displacement metrology of subnanometer and subpicometer resolution [6], [7]. The Fabry-Perot cavities applicated on femtosecond laser combs could leads to precise astronomical frequency calibration [8] or precision spectroscopy [9]

II. Theory

A. Femtosecond laser comb

Optical frequency combs are laser based on sources generating the femtosecond pulses. Train of the pulses are characterized by the central wavelength, period of pulses, pulse shape and pulse to pulse phase shift [1]. Train of femtosecond pulses produces a frequency spectrum of comb lines around the central frequency (wavelength):

$$f_i = f_{\rm ceo} + i \cdot f_{\rm rep} \tag{1}$$

where i is the number of comb lines in the order of 10^6 and $f_{\rm rep}$, and $f_{\rm ceo}$ are frequencies typically set in RF domain, called repetition and offset frequency respectively. The repetition frequency $f_{\rm rep}$ is indirectly proportional to period of pulses and the offset frequency $f_{\rm ceo}$ describe phase shift. These two frequencies are naturally stable only up to 10^{-7} and stabilization of all frequency comb lines is based on solution of two dimensional problem [10], [11]. Eq. (1) illustrate how are frequency comb lines sensitive to the stabilities of repetition frequency and offset frequency. The repetition frequency is very intensively present in RF spectra and could be then easily retrieved. Spectrum described by eq. (1) is typically broad only up to 100 nm (10 to 20 THz). For stabilization it is convenient to broadened to one optical octave where one could take lower frequencies (âĂIJredâĂİ) part, produce the second harmonic frequencies and interfere it with âĂIJblueâĂİ part of the comb spectra. Respecting the Eq. (1) and applying a simple math we could easily subtract the offset frequency:

$$2f_i - f_{2i} = f_{\rm ceo} \tag{2}$$

both $f_{\rm rep}$ and $f_{\rm ceo}$ could be stabilized by RF frequency standards, i. e. atomic clocks. The atomic clocks could theoretically have relitive stability in RF frequency domain down to 10^{-16} which could be in such way transfered to optical frequency domain. Here above mentioned method is so called **f-to-2f** or **self-referencing method** of stabilization. Another approach is based on stabilization of at least one of comb component to the very stable continuous wave (cw) laser source âĂŞ for ex. iodine stabilized Nd:YAG (with stability up to 10^{-14}) or He-Ne lasers.

B. Fabry-Perot cavity

Fabry-Perot cavity (FPC) consists of two mirrors separated by the distance $L_{\rm cav}$. FPC is instrument which transfer precise frequencies characterized from the optical path distance between mirrors and by the mirror reflectance. Two spherical mirrors with radius of curvature R_1 and R_2 transmitt the optical frequencies defined by:

$$\nu_{\rm opt} = j \cdot \nu = j \cdot \frac{c}{2 \cdot L_{\rm opd}}\,, \tag{3}$$

where c is speed of the light, j is integer number, ν is free spectral range (FSR) and $L_{\rm opd}$ is optical path distance between mirrors that is for confocal mirror configuration ($R_1 \cong R_2 \cong L_{\rm cav}$):

$$L_{\rm opd} = 2 \cdot nL_{\rm cav} \tag{4}$$

and

$$L_{\rm opd} = nL_{\rm cav} \tag{5}$$

otherwise, where n is refractive index of air between mirrors.

Frequency of m-th longitudinal mode transmitted through the non-confocal FPC can be expressed:

$$f_m = \frac{mc}{2nL_{\rm cav}} \tag{6}$$

Index of refraction and following dispersion bring us new problems in the further analyses. Let us for simplicity consider vacuum FPC. Periodical changes in distance between mirrors characterized by $L_{\rm cav}$ to $L_{\rm cav}+\Delta L_{\rm cav}$ (using for ex. the piezo-electric transducer.) with detector on output mirror produce according to Eqs. (3) and (6) transmission spectrum. Fluctuation of refractive index of ambient air changes the actual optical path distance and the transmitted optical frequency of light. Acustically and temperature stabilized chambers could be made to avoid any fluctuation. Vacuum chamber diminishes any fluctuation of ambient air according to the level of the refractive index. Special consideration have to be taken to account to avoid influence of ambient enviroment [12]. Stabilization of length between mirrors is made usually by motor or piezo-electric transducer.

III. Results

A. Pulse to pulse overlapping method

If

$$i \cdot f_{\rm rep} = j \cdot \nu \tag{7}$$

or in other words

$$f_{\rm cav} = x \cdot f_{\rm rep} \tag{8}$$

where x, i, and j are the integers. Each optical frequency comb line could be transferred through the FPC but the optical path distance between the mirrors should strictly respect following equation

$$L_{\rm cav} = \frac{c}{2xf_{\rm rep}} \tag{9}$$

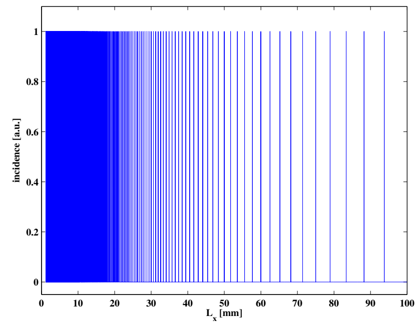

Fig. 1. Places of incidence between spectral components of the optical resonator and mode-locked laser vs. resonator length L_x. The calculation is done for the mode-locked laser with the repetition frequency $f_{\rm rep}$ = 100 MHz. The refractive index in the cavity is not considered

In frequency domain it means that laser spectral comb lines coincide with transmission of the FPC. Fig. 1 represents a structure of such an optical path distances which correspond to an optical frequency comb with $f_{\rm rep} = 100$ MHz.

The structure of the problem in the time domain is more complicated since in typical FPC we suppose continuous wave sources targeting the mirrors. In our case the source has in the simplest a structure of âĂIJcontinuous wave sourceâĂİ central wavelength modified by the pulse repetition and pulse shaping. Strictly, the pulses have no central wavelength but they could be represented by the number of wavelengths sources. Pulses are in the time domain explained by [10].

$$\mathrm{E_p}(t) = \mathrm{E_{env}}(t) \cdot e^{i(\omega_{\rm c}t+\phi_{\rm ce})} \tag{10}$$

where $\mathrm{E_{env}}$ is envelope function of the pulse and $\omega_{\rm c}$ and $\phi_{\rm ce}$ are the carrier angular frequency and carrier-envelope phase which is for i-th pulse given by $\phi_{\rm ce} = i \cdot \Delta\phi_{\rm ce} + \phi_0$. The train of y pulses travelling from the frequency comb source is described by following relation

$$\mathrm{E}(t) = \sum_y \mathrm{E_p}(t - y \cdot \tau) \tag{11}$$

where $\tau = 1/f_{\rm rep}$. Relation between time and frequency domain quantities is covered by

$$f_{\rm ceo} = -\frac{\Delta\phi_{\rm ce} \cdot f_{\rm rep}}{2\pi} \tag{12}$$

Pulses travels through Fabry-Perot and i-th pulse meets $(i+1)$-th pulse after j round-trips. This simple assumption is strictly fulfilled in totally evacuated medium. The effect of dispersion and carrier-phase should be taken to acount in resulting interference of pulses inside the cavity. If one pulse overlap through the next in the FPC they interfer producing a structure of maxima and minima âĂŞ the interference fringes which envelope corresponds to the pulse shape and pulse length. Increasing the number of incidence pulses and the quality of

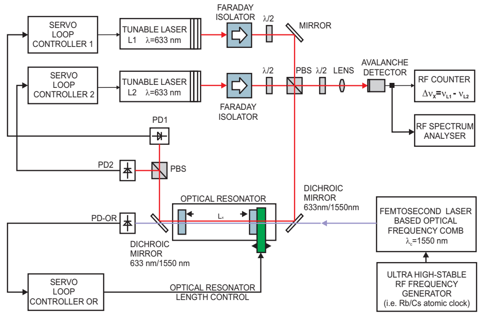

Fig. 2. The schematic diagram.$\lambda/2$ are half-wave plate for 633 nm, L1 and L2 are He-Ne lasers used for measurement of inter-mode frequency $f_{\rm cav}$, and OR is abreviation of the optical resonator.

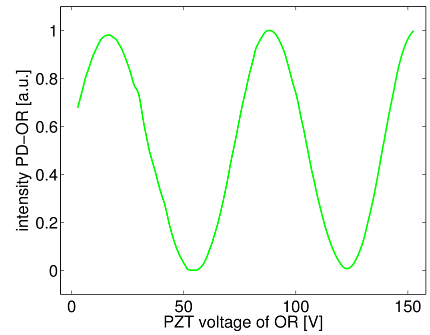

Fig. 3. The record of of the interference of femtosecond laser on the output of the optical resonator.

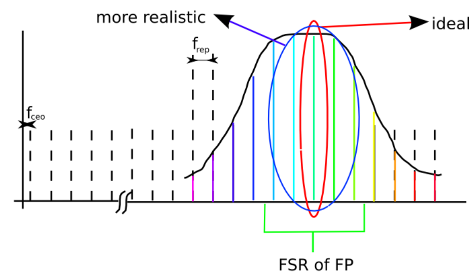

Fig. 4. Schematics of optical grating separation of comb teeth and DFB diode at the similar wavelengths. FSR is ideally chosen Fabry-Perot resonator free spectral range.

the Fabry-Perot cavity results in better contrast around the well defined precise length (according to the Fig. 1). The precise length is at centroid of resulted interferogram.

The schematic set-up of the whole optical and electronic parts is in Fig 2. This experiment were made with the femtosecond mode-locked laser based optical synthesizer working at the central wavelength at 1550 nm (ranging from 1500 to 1600 nm) and with $f_{\rm rep} = 100$ MHz. The output power of the femtosecond laser in the whole spectra was around 40 mW. The cavity of the optical resonator was designed in the plan-parallel configuration. The two He-Ne laser beams (lasers L1 and L2) were separated by different polarization. We used polarizing beam splitters PBS and set of half-wave plates for this purposes. For locking of the optical frequency of each laser L1 and L2 to neighboring longitudinal modes of the optical resonator we used a harmonics detection technique. On basis of places of incidence shown in Fig. 1 we selected the length of the cavity of the optical resonator $L_{\rm cav}$ to 375 mm approximately. The laser beam is expanded in collimator to 4 mm diameter to optimal Gaussian propagation mode structure with mode TEM_{00}. Higher transversal modes eliminates by the slit in the center of the cavity. After the locking of both lasers L1 and L2 to neighboring longitudinal modes we tuned the length of the cavity $L_{\rm cav}$ precisely with respect to possible range of repetition frequency of the mode-locked laser $f_{\rm rep} = 100$ MHz (+129 kHz,-257 kHz). We reached the free spectral range $\nu = 399.7500$ MHz by very precise tuning of the cavity length $L_{\rm cav}$. The reflectivity was 99% at 633 nm and about 90% for central wavelength of fs comb by TiO_2/SiO_2 structured film mirrors on glass substrate. The pulse to pulse overlapping recorded by scanning FPC is presented in the Fig. 3.

B. Comb line separation

First approach to put fs pulses directly into the cavity has problem of idenfication of precise length to which the cavity must be locked [13]. Another approach is separation of some wavelengths of the comb spectra by a diffraction grating.

The scheme of the comb teeth separation is in the Fig. 4. In an ideal case one comb frequency (tooth) is separated through well chosen diffraction grating. In the case of femtosecond laser comb with 100 MHz repetition frequency at 1550 nm it must separate only 0.0008 nm. In free-space optics the nature of reflective holografic grating could separate around 0.5 nm in the infrared region. The problem increases than the FSR of the resonator is smaller than the diffraction grating separating the spectra of femtosecond comb. In this case the resonator optical spectra on the detector do not represent one comb line to one detected lines but number of overlapped lines.

The experimental set-up for the method is same as in Fig. 2 except the separation of the comb lines as sketched in Fig. 5. The coarse distance between mirrors was found by using two He-Ne polarization separated lasers [14]. The recordings of the comb lines was done for the 400 MHz Fabry-Perot resonator with two mirrors in planparallel geometry (Fig. 6) and for 2 GHz Fabry-Perot resonator in confocal geometry (Fig. 7). The piezo-scanning voltage range could transmit more one FSR of the resonator so we could observe up to 5 lines in Fig. 6

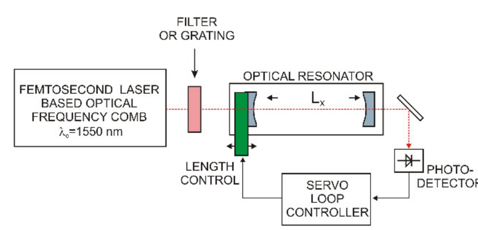

Fig. 5. Placing of the optical frequency filter: grating and slit, into the set-up.

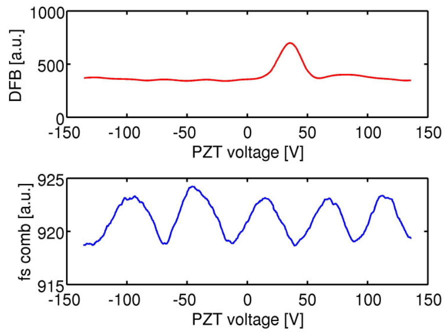

Fig. 6. Recordings of the intensity on the infrared optical detector from DFB diode and from the femtosecond comb in the Fabry-Perot resonator with FSR of 400 MHz.

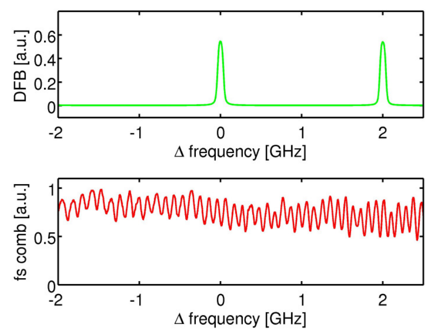

Fig. 7. Recordings of the intensity on the infrared optical detector from DFB diode and from the femtosecond comb in the Fabry-Perot resonator with FSR of 2 GHz.

and more than 20 lines in Fig. 7. JDS Uniphase CQF935/708-19440 DFB diode operated at central wavelength 1542.14 nm (194.40 THz) with central linewidth 1 MHz and wavelength tuning range of 1.8 nm and maximal wavelength drift with case temperature 0.001 nm/°C. The DFB diode was not locked to any of the frequency comb line but serves as reference for optical comb spectra.

The precise length is defined by the each transmitted line in Figs. 6 and 7 in the case of narrow spectra separation by diffraction grating. The recordings signal-to-noise ratio and bad accuracy at the center of the recorded line makes such locking complicated. In our experiment we used the ruled diffraction grating with the selectivity 0.56-1.46 nm/mrad (600 and 1200 lines/mm). Our femtosecond comb produces the comb lines in the range from 1500-1600 nm such as 124914 components. The distance from the grating to the slit was from 350 mm to 500 mm and the spectra was cut by the slit closing the diffracted beam in the range from 1 mm to 10 mm width. The narrowed output spectra still have around 1/100 (around 1000) of the components according to the configuration. The reflectivity of mirrors was better than 99% in the range from 1500 to 1600 nm. The finesse of the cavities were up to 330 for our optical region and the natural linewidth of resonators was more than 3 MHz.

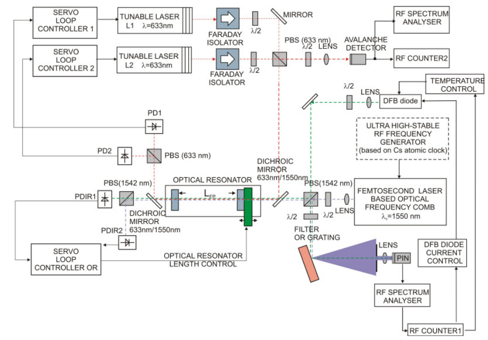

Fig. 8. The scheme of the optical and electronic arrangement with free space diffraction grating. PD1, PD2, PD-OR are photodetectors, $\lambda/2$ are half-wave plate for 633 nm and for 1550 nm, L1 and L2 are He-Ne lasers used for coarse measurement of FSR, and OR is abreviation of the optical resonator.

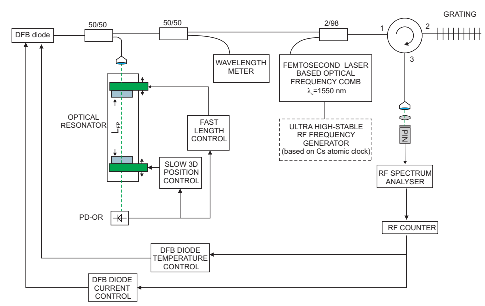

Fig. 9. Experimental set-up. Optical set-up is made in fiber optics. DFB diode laser light is splitted twice by 50/50 splitter: a) to Fabry-Perot cavity, b) to wavelength meter. The rest of the light is mixed with the femtosecond fiber lasers by 2/98 coupling. The interference between DFB laser output interfere with femtosecond laser comb at circulator with the fiber Bragg grating and output at the PIN photodetector. Beat frequency is used to temperature and current control of the DFB diode. The lenght of the FP cavity is controlled by 3 slow coarse positioning PZT's and 1 fast fine PZT for length control.

C. DFB diode lock

Next step to the stabilized FPC is through DFB diode frequency locked to the closest optical frequency comb line and only the light of DFB diode is transmitted through FPC according to scheme in the Fig. 8. The problems with free space optics, stability, and of wide grating separation spectra solves using of fiber optics. Scheme of such experimental set-up is in the Fig. 9.

Optical part is coupled in single mode fibers with E2000 and FC/APC connectors. Case temperature of the diode was controlled by temperature controller and locked to the 0.001°C. Maximal optical output power of DFB diode at FC/APC fiber is 40 mW. The DFB diode laser light is transmitted to the Fabry-Perot cavity and the part of the light is directly connected to the fiber input of Highfinesse, GmbH wavelengthmeter WS Ultimate 30LIR with absolute accuracy down to 30 MHz. The measurement resolution gives us coarse information about the DFB position with respect to the femtosecond laser comb line, each separated by 100 MHz. The rest of the DFB laser diode light (25%) is mixed in fiber coupler in mixing ratio 2% of the light versus 98% of the light of femtosecond laser comb. The output of 2/98 coupler between femtosecond fiber laser and DFB diode laser is coupled to fiber circulators input 1. The light comming from input 1 was separated in the circulator by two different fiber Bragg gratings made by O/Eland, Inc. at 1541.476 nm and FWHM of 0.198 nm and reflectivity of 85.07% and at 1540.430 nm and FWHM of 0.114 nm and reflectivity of 97.04%. The beat signal is detected at the output 3 by Menlosystems, GmbH free-space high sensitivity fast PIN photodetector FPD-510-F with fiber output and the collimator. The beat signal between the DBF laser diode and closest comb line is detected by Hewlett Packard 8560E spectrum analyzer conected through GPIB to UBS connector to the computer for computer control in Labview. The Labview program records and controls the beat frequency by the temperature controller and the diode current controller loops.

The Fabry-Perot cavity consisted of two mirrors with radius of curvarture 148 mm separated in distance by roughly 148 mm with reflexivity of 99.08%. The length separation between mirrors is controlled by 4 piezotranducers (PZT). Fast fine PZT lift of 0.765 μm controls the length of the cavity by bipolar ($\pm$ 150 V) cylindrical piezo tube fixed in KC1/M Thorlabs mirror mounting with 3 unipolar PZT with translation range 4 μm each. Kinematics mounts distance is fixed by four ER8 cage rods with 6 mm in diameter.

IV. Conclusion

We experimentally demonstrated three methods for generation of precise length. The first method with strictly specified mirror distance with planparallel geometry is very sensitive to the mirror alignment. Its main disadvantage is that the distance should be locked only to some mirror distance represented by discret length spectrum. The second method based on separation of frequency comb lines produce illimitable variety of length to be locked on. The resonator length is independent to the laser comb repetition frequency. Nevertheless it is very demanding on the grating selectivity. Third method is based on locking of DFB laser diode to closest fs comb line. It seems to be very promising. It combines relatively intense laser source detection represented by DFB diode and very good frequency stability represented by the femtosecond laser comb.

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